What you should have learned from Recitation 3

Fei Qi

Rutgers University

fq15@math.rutgers.edu

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- The slides are intended to serve as records for a recitation for math 244 course. It should never serve as any replacement for formal lectures or as any reviewing material. The author is not responsible for consequences brought by inappropriate use.
- There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.

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Exercise: Check that $u_3(t) = T$ is a solution of the differential equation $u' = -k(u - T)$.

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(Where does u_3 come from? When you get u_1, u_2, you assumed $u \neq T$ so $1/(u - T)$ makes sense. $u_3(t)$ arises when you remove this assumption)

Exercise: Check that $u_3(t) = T$ is a solution of the differential equation $u' = -k(u - T)$.

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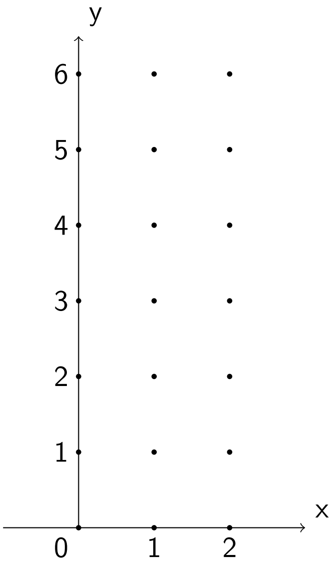
The direction field in the quiz



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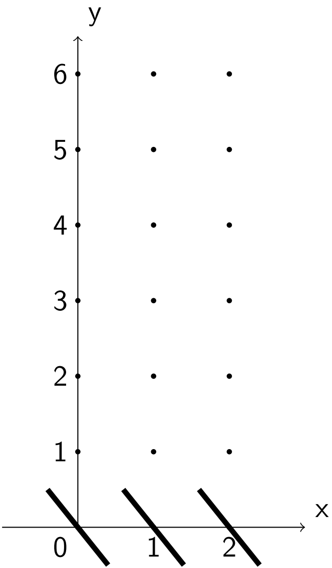

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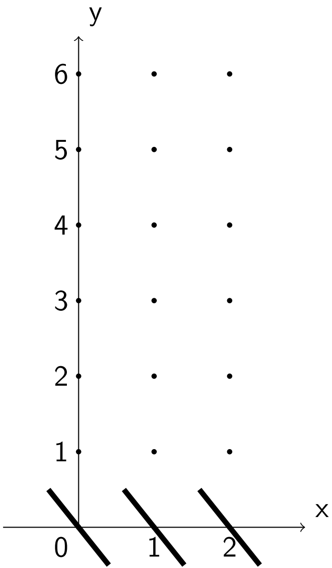

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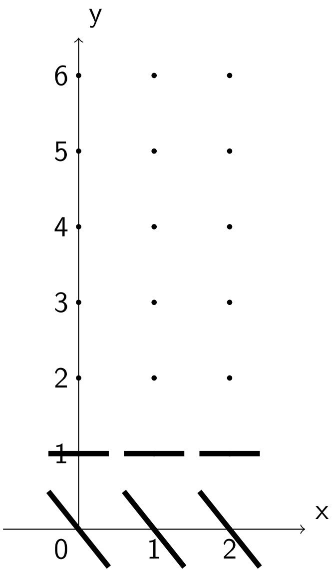

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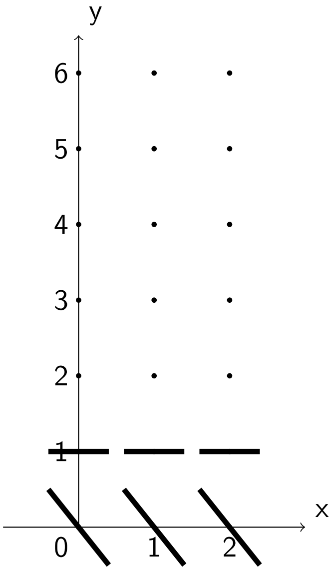

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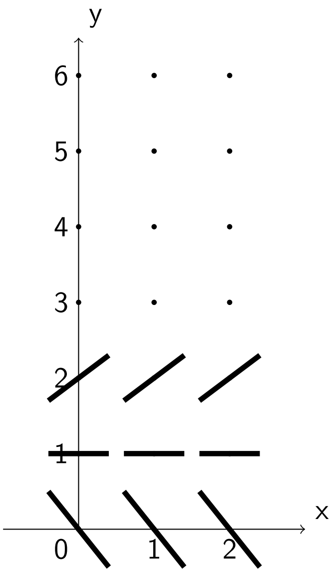

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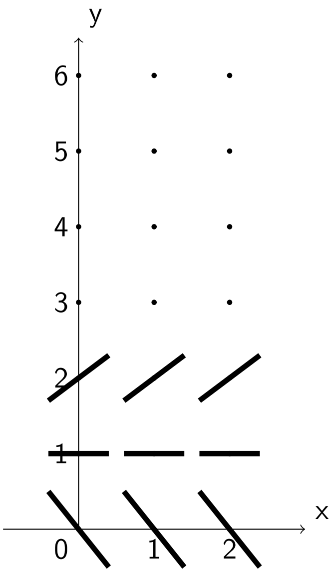

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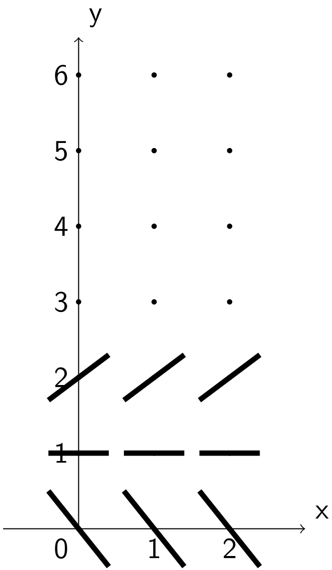

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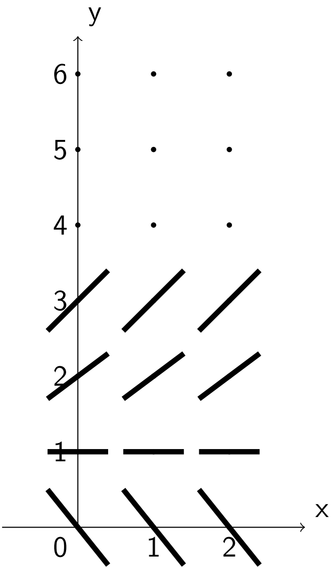

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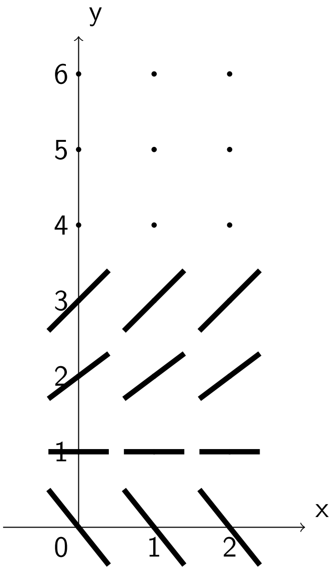

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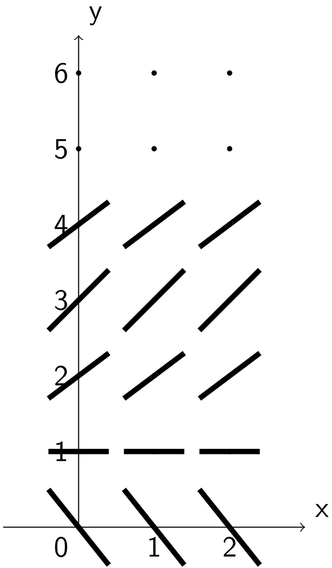

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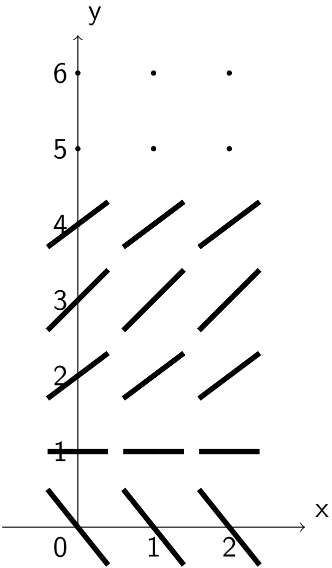

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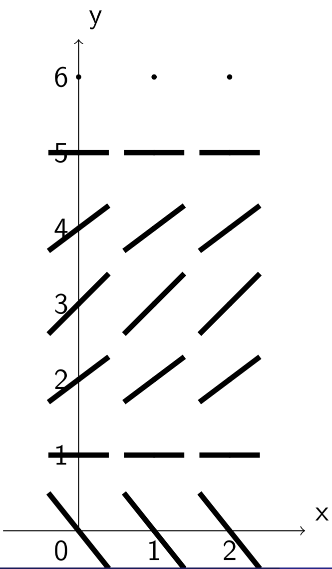

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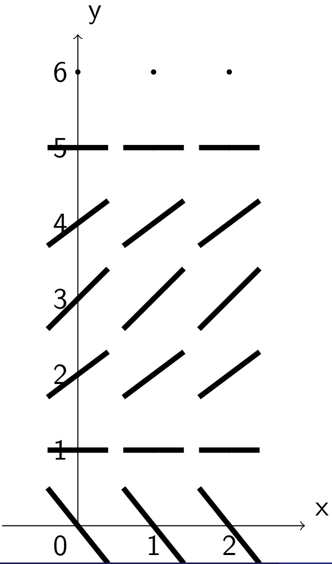

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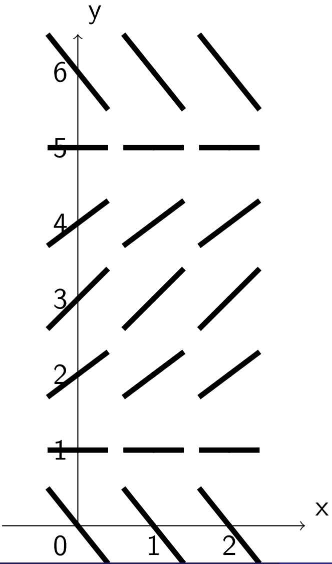

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- The direction fields tells you how solutions behave WITHOUT REFERRING TO WHAT THE SOLUTION IS. Sometimes even the solutions don't tell you that.
- You can draw direction fields for any first order equations $y' = f(x, y)$, no matter how ugly $f(x, y)$ is. That's why direction fields are useful.

For interested students who wants to see how to draw direction fields by hand for non-autonomous ODEs, please watch MIT Lecture 1, 13:49 to 20:11 for general guidelines and 20:11 to the end for examples.

The End